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MAR 79 W P OLSON, K A PFITZER, G J MROZ N00014-78-C-0215

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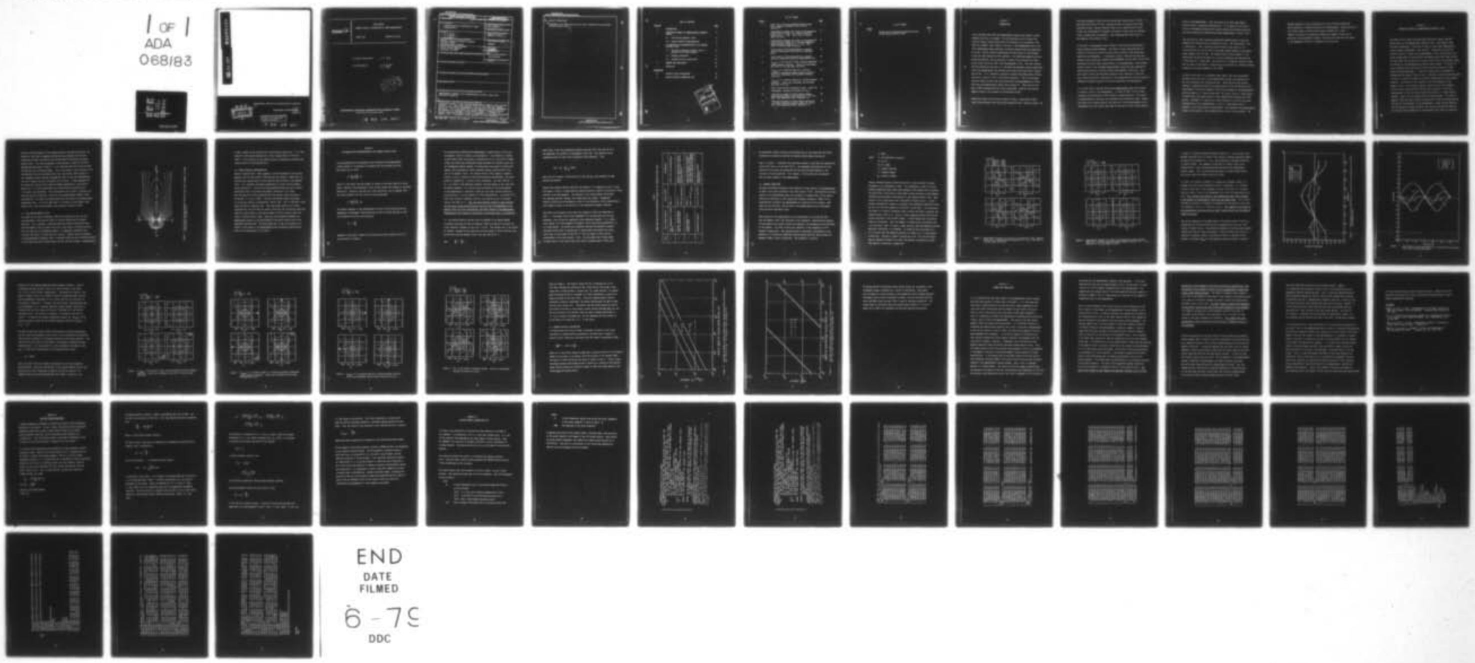

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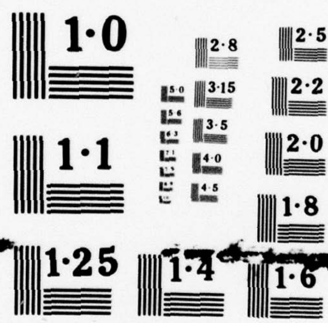

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FINAL REPORT

CHARGED PARTICLE ACCELERATION IN INNER MAGNETOSPHERE

MARCH 1979

N00014-78-C-0215

Principal Investigator: W. P. Olson

Co-Investigators: K. A. Pfitzer
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A quantitative model of the disturbed magnetospheric magnetic field has been developed. In it the fields produced by each of the three major magnetospheric current systems (magnetopause, ring, and tail) have been represented separately. This is necessary since during "magnetically active" periods the strengths and topology of these currents change. A "template" of a magnetic storm was developed in order to represent the associated time varying magnetic field. The magnetic vector potential was also represented. A

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20. Abstract (Continued)

knowledge of its time history can be used to determine the associated induced electric field.

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Section 1

INTRODUCTION

It has long been known that the magnetosphere contains many regions in which energetic particles are trapped. The Van Allen belts are, of course, the classical example containing particles with energies up to and beyond 1 Mev. There are, however, other regions in the tail of the magnetosphere and in the inner magnetosphere where additional relatively energetic particle populations are found. Several detailed explanations have been given for the existence of the very high energy Van Allen radiation. The remaining energetic particle populations, however, must be explained in terms of the interaction of their source region, the solar wind, with the geomagnetic field. The solar wind has particle energies on the order of 1 keV while the energetic particle populations within the magnetosphere have energies which range from several keV to several tens of keV. It is, therefore, necessary to explain the energization of these particles in terms of "local" acceleration mechanisms. That is, the energization of these particles must take place within the magnetosphere or in the blow shock and magnetosheath regions which surround it. There are only two ways in which charged particles can be accelerated: either by time varying magnetic fields or the presence of electrostatic fields.

Generically there are two kinds of electric fields: electrostatic fields result from processes which cause charge separation over a region of space, and

time varying magnetic fields which have associated induced electric fields. In describing both types of fields, care must be taken in selecting the proper coordinate system. Because reference frames are so important with electric fields, the transformation of magnetic and electric fields from one reference frame to another must be considered. Such transformations often result in the presence of the electric field.

In the earth's magnetosphere several electric fields have been postulated to explain observed plasma phenomena. The electric fields associated with the rotation of the earth are used to explain the corotation of the plasma which populates the plasmasphere. A "convection" electric field arising from some unknown viscous interaction between the magnetosphere and the solar wind has been postulated to explain gross plasma motions in the magnetosphere. More recently electric fields in the ionosphere-magnetosphere system have been suggested to explain the existence of currents flowing along magnetic field lines. Thus, the electric fields that have been discussed in the literature to date have been linked with specific observations in the magnetoshere but have not typically addressed the general question of particle acceleration and energization.

It is useful then to consider what we know <u>quantitatively</u> about the existence of electric fields in the magnetosphere. It turns out that little is known concerning the electrostatic fields caused by charge separation. The subject of induced electric fields has also received little attention to date. However, the source of an induced electric field must be a time varying magnetic field. Considerable work has recently been done on the nature of time varying magnetic

fields in the magnetosphere. Thus, the source of at least some induced electric fields is understood quantitatively. It is because of this availability of information concerning the source functions that we have during the past year quantitatively examined the induced magnetospheric electric field.

From the point of view of particle energization induced electric fields are somewhat more interesting than electrostatic fields. The electrostatic field is conservative. Thus, a particle moving through it upon return to its initial position will end up with no net energy change although it may undergo energy changes along its path. The induced electric field, however, is not conservative and particles moving in it can be accelerated and energized even if they move over a closed path. The selection of a convenient reference frame, and its impact on the electric fields which can be observed in rotating reference systems are discussed elsewhere in this report.

The goal of this work is to ultimately shed light on the local acceleration processes which must exist in the magnetosphere and that are responsible for the local energization of charged particles. During the past year we have examined in quantitative detail the induced electric field associated with the daily wobble of the earth's magnetic field and the response of the magnetospheric current systems to it. We have found that this source, which is always present, is smaller than the induced electric field expected during substorm and storm periods. Although it is found to be a small fraction of a millivolt per meter throughout most of the magnetosphere, it still is large enough to be of interest in the study of particle energization. The response of the magnetospheric plasma to the presence of this induced electric field is also discussed. The

response mechanism is very complicated and in fact different mechamisms may be operative in different regions of the magnetosphere. Before discussing this induced electric field and the plasma's response to it, some comments are made on the quantitative modeling of magnetic fields and in particular the magnetospheric magnetic field resulting from the daily wobble of the geomagnetic field as it responds to the solar wind.

Section 2

QUANTITATIVE MODELS OF MAGNETOSPHERIC MAGNETIC FIELDS

The authors of this report have developed with partial support from ONR several models of the magnetospheric magnetic field. These models differ from their predecessors in that they include all three major magnetospheric current systems; magnetopause, ring, and tail. Because the ring and tail currents flow over a volume of the magnetosphere, it was required that a procedure be developed for quantitatively representing such distributed currents. Previous models had used only point sources (dipole field) and currents flowing on surfaces (on the magnetopause and neutral sheet regions). The details on how to represent these distributed currents have been discussed in other reports and published papers. It was also required that a new method for representing the magnetic field be found. Previously, models had been described in terms of a spherical harmonic series which represented the scalar magnetic potential. Derivatives of this potential yield the components of the magnetic field. This formalism, however, cannot be used in the presence of the currents since the field from the scalar potential is curl free. A representation of the field which allowed the presence of a finite $\nabla \times \bar{B}$ was required. Details of this problem have also been published. The resulting magnetic fields have been tested by comparing their output with many observed particle and field features in the magnetosphere. These include comparison with ΔB contours of Sugiura and Poros, the correct calculation of observed low latitude cutoff boundaries for precipitating solar cosmic ray particles,

the high latitude boundary of the trapping region of low energy particles, the topology of the field as compared with barium cloud releases, and detailed comparisons of model calculations with the observed field measured at geosynchronous orbit. The resulting magnetic field topology is shown for the noon-midnight meridian plane in Figure 1. It is noted that the field lines are much less dipolar than in previous models. This is the result of the inclusion of the contributions from the distributed quiet time ring current, which because of its diamagnetic nature depresses the magnetic field in the inner magnetosphere. This was an output from our first model in this series. It was developed with the restriction that the solar wind be incident perpendicular to the earth's dipole axis. In a more recent model this restriction was relaxed and all "tilt angles" were permitted. The resulting model can be used to represent the daily and seasonal variations in the quiet time magnetospheric magnetic field configuration. This model was tested extensively using quiet time geosynchronous magnetometer data. It is this model that is of interest to the present study because such a time varying magnetic field produces an electric field.

2.1 TIME VARYING MAGNETIC FIELD

As pointed out by Hones and Bergeson (1965), the existence of only one time varying magnetic field does not result in the net energization of charged particles. This is because a reference frame can always be chosen that moves with the magnetic field variation such that in that frame no variation persists. Thus, for the case of the wobbling dipole, in a geographic reference frame (in which the dipole is fixed) $\partial B/\partial t$ will be zero. However, when the response of the magnetospheric magnetic field is included, there can be no coordinate system in which both the earth's dipole motions and the resultant changes in magnetospheric

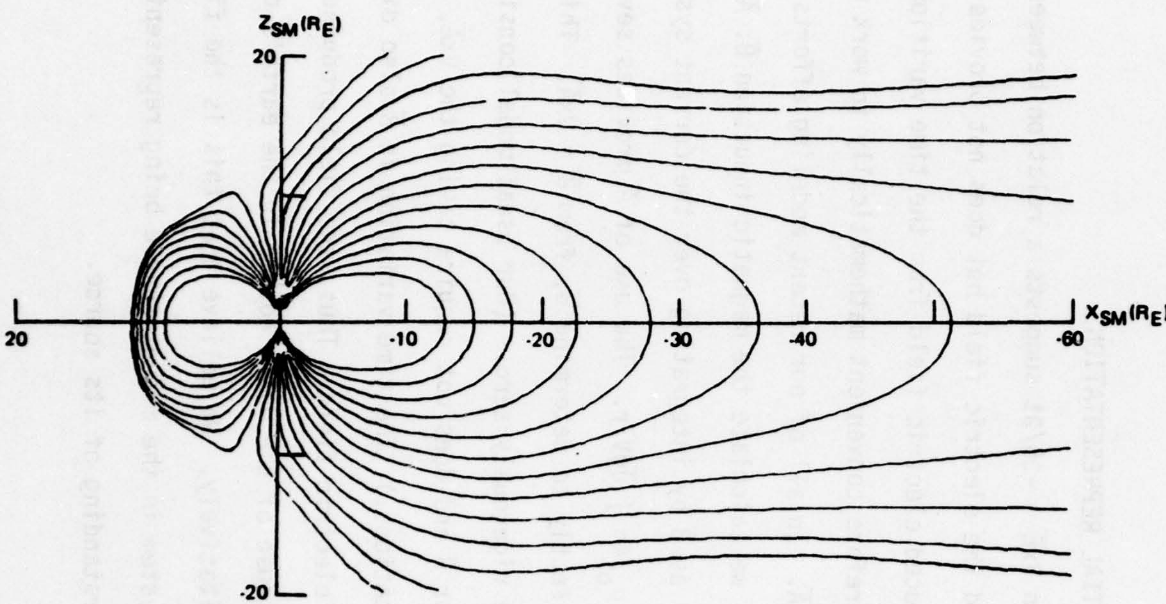


Figure 1. Field line in the noon-midnight meridian plane from a magnetic latitude of 70° to 90° in steps of 2°.

current systems can be cancelled out such that $\partial B/\partial t$ equals zero. It is thus expected in the present problem that a finite induced electric field will result. It was the goal of the current period of performance to describe this induced electric field quantitatively.

2.2 VECTOR POTENTIAL REPRESENTATION

Maxwell's equation $\nabla x \overline{E} = -\partial B/\partial t$ suggests a relation between the time varying magnetic field and the electric field but does not provide a simple means for obtaining the induced electric field from the time varitions in the magnetic field. It is therefore convenient mathematically to work with the magnetic vector potential, $\overline{A}$. In all of our recent modeling efforts we have routinely determined $\overline{A}$ when we calculate the magnetic induction $\overline{B}$. $\overline{A}$ is found much the same way as $\overline{B}$ by integrating over the current system according to the formula $\overline{A} = \mu_0/4\pi \int \overline{J}dV/r$. The use of $\overline{A}$ provides several advantages. It can be used directly to determine $\overline{B}$, from $\overline{B} = \nabla x \overline{A}$. This is required in cases where $\nabla \cdot \overline{B}$ must be rigorously zero. (Our usual model consists of a direct representation for $\overline{B}$ and does not require taking the $\nabla x \overline{A}$, eliminating additional numerical complications.) The time variation in $\overline{B}$ also directly yields the value of induced electric field. Thus, the field produced throughout the magnetosphere because of the daily wobble in the earth's dipole axis can be determined quantitatively. We believe that this is the first example of an electric field system in the magnetosphere being represented quantitatively with a firm understanding of its source.

Section 3

THE QUANTITATIVE REPRESENTATION OF THE INDUCED ELECTRIC FIELD

In the determination of the magnetic field produced by the magnetospheric current systems it is necessary to integrate over the currents using the Biot-Savart law to yield

$$\overline{B} = \frac{\mu_o}{4\pi}\int \frac{\overline{J}x\overline{r}}{r^3}\, dV$$

where $\overline{r}$ is the vector from the element of current to the point where $\overline{B}$ is being found. Because of the complexity of this current system, the integral is performed numerically. Similarly, and with very little additional cost in computer time, the vector potential for this field can also be obtained,

$$\overline{A} = \frac{\mu_o}{4\pi}\int \frac{J}{|r|}\, dV$$

The induced component of the magnetospheric electric field resulting from this dependence of the magnetic field explicitly on the tilt angle and thus on time can be obtained from the vector potential

$$\overline{E}_I = - \frac{\partial\overline{A}}{\partial t} .$$

(Appendix A describes a method for calculating the vector potential for $\overline{A}$ of any arbitrary tilt angle.)

This procedure for obtaining the magnetospheric induced electric field (and the magnetic field) is actually an approximation. The existence of a medium in which these fields are present is considered only as a source ($\overline{J}$) of these fields; the fields have traditionally been considered as virtually static and all propoagation effects ignored. In regions where the plasma is particularly tenuous this approximation yields satisfying results, particularly in the case of the magnetic field. The induced electric field, however, presents a problem, in that in addition to the vector potential contributions, the scalar potential component $\overline{E}_s = -\nabla\phi$ must also be considered. The total field $\overline{E}_T = -$ $\overline{E}_T = -\nabla\phi - \partial A/\partial t$ is the observable quantity, and both aspects of the field must be properly modeled. It is certain that the presence of an ionized medium in the magnetoshere is important to $\partial\overline{A}/dt$ but that under certain conditions, discussed below, it can be ignored to a first approximation. In the case of $\overline{E}_s$ (the scalar potential component) both the local medium as well as extended regions contribute to it. <u>Thus, the total electric field at a given location will not only include the contribution from the local time dependence of the magnetic field, but also contributions from other regions of the magnetosphere transmitted to this location by the medium, to the extent that it is conductive.</u>

3.1 THE SCALAR POTENTIAL ELECTRIC FIELD IN PRESENCE OF AN IONIZED MEDIUM

A procedure developed by Hones and Bergeson (1965) was applied to evaluate the scalar potential component of the elect^ic field. They assumed that in the region of interest, charged particle densities are high enough to assure sufficient conductivities along the magnetic field lines such that $\overline{E}_T \cdot \hat{B} = 0$.

Then $$\frac{d\phi}{ds} = \frac{\partial A_{\parallel}}{\partial t} ,$$

where $d\phi/ds$ is the scalar potential gradient along the field line, and $\partial A_{\parallel}/\partial t$ is the component of $\overline{E}_I$ parallel to the magnetic field line. This equation can be integrated along the field line to yield the scalar potential. Thus,

$$\phi(r) = \phi_B - \frac{\partial}{\partial t} \int_{FL} \overline{A} \cdot \overline{ds} \quad ,$$

where the line integral is along the field line, and ϕ_B is the potential at some appropriate boundary.

Because this approach greatly simplifies the problem, it is tempting to use it, along with models of $\partial\overline{A}/\partial t$, to quantitatively determine possible sources of the electric field observed in the magnetosphere. The presence of the ionized medium, which makes this approach possible, however, also complicates the problem. Propagation effects on $\partial\overline{A}/\partial t$ must be considered, as well as the shielding of the $\overline{E}_S$ contributions which depend strongly on the selection of the appropriate boundary condition.

The effects of the medium on the field line integral of $\partial\overline{A}/\partial t$ are summarized in Table 1. The simple case of time independent $\overline{B}$ is included here only for completeness, since it does not represent a very interesting example. It is consistent, however, with the assumption of magnetic field lines being equipotentials in an ionized medium. The second case represents the Hones and Bergeson procedure (discussed above) which is appropriate in regions where the conductivity along field lines is high enough so that $\overline{E} \cdot \overline{B} = 0$, but low enough to allow nonlocal contributions to the total electric field. In the extreme case of high conductivities (Case 3) along field lines, only locally induced electric fields need

Table 1

CONDITIONS FOR SELECTIONS APPROPRIATE VALUES OF THE INDUCED ELECTRIC FIELD

Case	Gross Plasma Features	Value of $\partial\overline{A}/\partial t$	Consequences
1	Time independent $\overline{B}$, ionized medium	$\partial\overline{A}/\partial t = 0$	Field lines are equipotentials
2	Tenuous, ionized medium; very low conductivities	$\partial\overline{A}/\partial t \neq 0$ $\overline{E}\cdot\overline{B}=0$ and $d\phi/ds=\partial A_{\parallel}/\partial t$	Nonlocal contribution to total electric field; field lines are <u>not</u> equipotentials
3	Ionized medium; high conductivities	$\partial\overline{A}/\partial t \sim 0$ $d\phi/ds = \partial A_{\parallel}/\partial t \big\|_{local}$	Local contributions to electric field only; external sources cannot contribute due to propagation effects (energy dissipation)
4	Nonconducting medium	$\partial A_{\parallel}/\partial t \neq 0$ and $\overline{E}\cdot\overline{B} \neq 0$	Approximate procedure inappropriate

be considered; external sources can contribute only to the extent that the fields originating from them can penetrate the medium without energy dissipation.

Case 4, in Table 1, represents the conditions present in areas whre the conductivity along field lines is so low that $\overline{E}\cdot\overline{B} \neq 0$. The procedures discussed here for the solution of $\overline{E}$ are then not appropriate and the only available approach is the solution to the wave equation. This, however, is the vacuum case and probably does not apply anywhere in the magnetosphere.

3.2 BOUNDARY CONDITIONS

The Hones and Bergeson procedure can be applied to large regions of the magnetosphere to obtain first order estimates of the total electric field. Contributions from those regions of the magnetosphere where this procedure is inappropriate, can be approximated by the selection of other appropriate boundary conditions. Models of ion concentrations throughout the magnetosphere are required for this purpose. When these are not available, limited areas of the magnetosphere can be investigated using available experimental data.

Other regions of the magnetosphere can be investigated, by utilizing the fact that the magnetic field lines terminate in the ionosphere. Neglecting the presence of high conductivity regions outside the ionosphere, the ionosphere can be considered as the boundary. The value of the scalar potential in the ionosphere is still subject to speculation. One candidate based on theoretical consideration is the potential of a uniformally magnetized rotating sphere which was used by Hones and Bergeson (1965) in their calculations. This potential is given by

$$V = \overline{U} \cdot \overline{A} \quad ,$$

where V = the potential at point $\overline{r}$,

$\overline{U} = \overline{w} \times \overline{r}$,

$\overline{A} = \overline{\mu} \times \overline{r}/R_s^3$,

$\overline{w}$ = angular velocity

$\overline{\mu}$ = magnetic moment

R_s = radius of sphere.

Another candidate boundary condition would be to simply consider the upper ionosphere as an equipotential region. This assumption, as well as the uniformally magnetized sphere model were employed in the present work to evaluate the sensitivity of the electric field model to both the choice of the boundary condition and the nonlocal contributions to the induced electric field. Some results of this investigation are illustrated in Figures 2 and 3. In both figures, equipotential contours are plotted in the geographic equatorial plane. The sun is to the left (negative x-direction) in both plots. The calculations were done in an inertial coordinate system in which the earth is rotating, the induced electric field (i.e., $\partial A_{\parallel}/\partial t$) is due to both the dipole field and the diurnal variations in the current systems. In both figures four calculation times are illustrated: thus, for example, in Figure 2a the calculation time is UT (universal time) = 0.0 hours, summer solstice, when the Greenwich meridian (positive x-direction) is at midnight. In Figure 2b the calculation is at UT = 6.0 hours when the Greenwich meridian is at dawn (i.e., in the positive y-direction). Similarly, Figures 2c and 2d are at UT = 12.0 and 18.0 hours when the Greenwich meridian is at noon (the negative x-direction) and dusk (the negative y-direction), respectively.

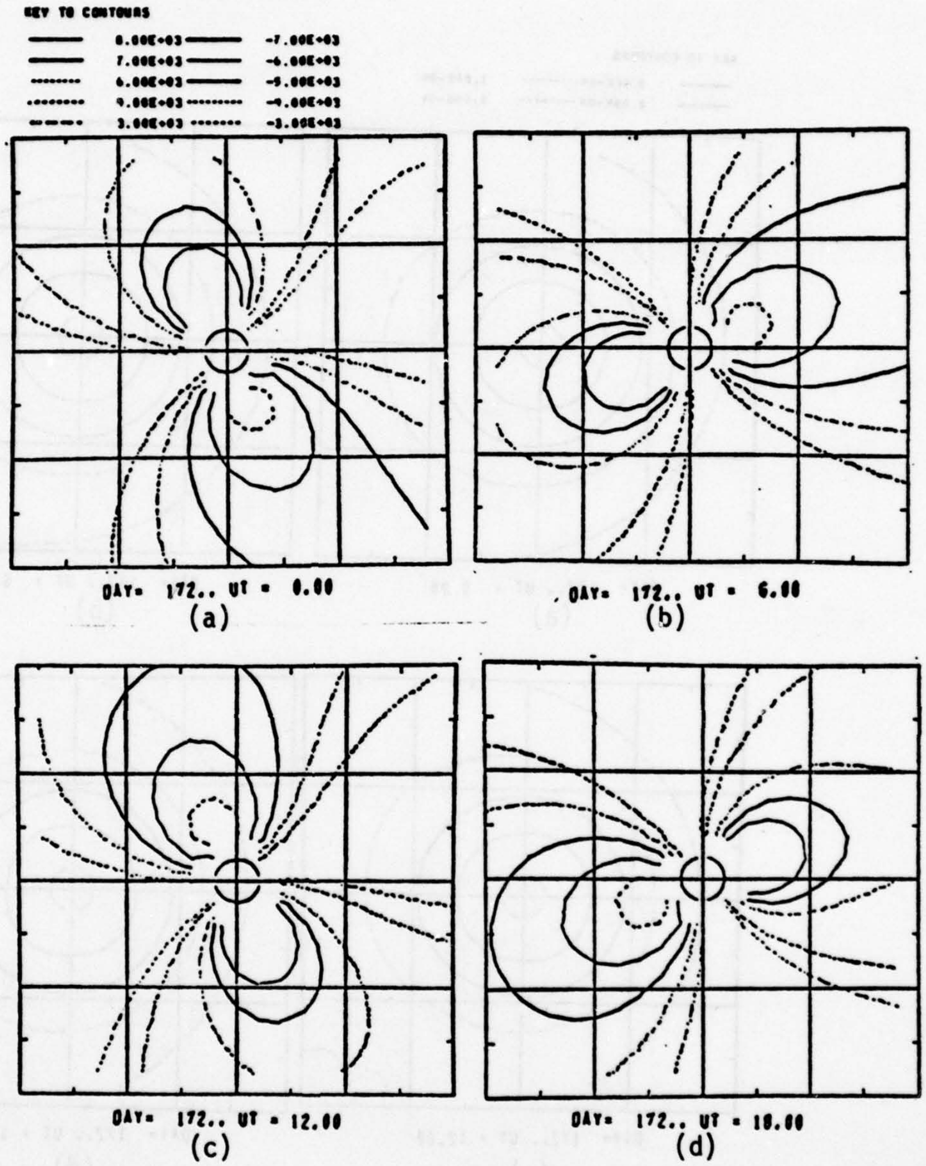
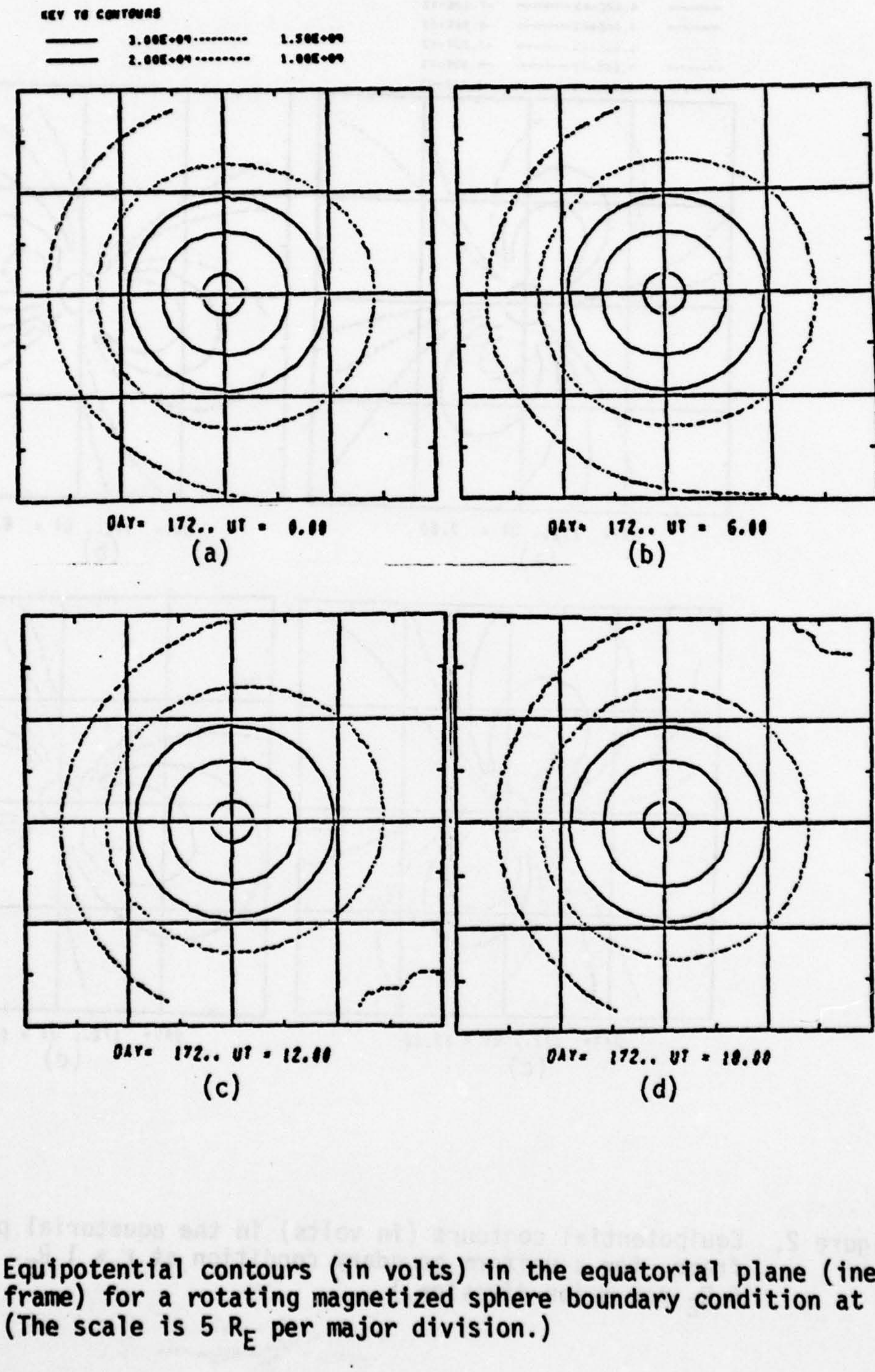


Figure 2. Equipotential contours (in volts) in the equatorial plane, inertial frame, for a uniform boundary condition at $r = 1\ R_E$. (The scale is $5\ R_E$ per major division.)



(a) (b) (c) (d)

Figure 3. Equipotential contours (in volts) in the equatorial plane (inertial frame) for a rotating magnetized sphere boundary condition at $r = 1\ R_E$. (The scale is 5 R_E per major division.)

In Figure 2 a uniform (constant) potential surface at $r = 1\ R_E$ was used as the boundary condition; while in Figure 3 the rotating, uniformly magnetized shphere was used at the boundary of $r = 1\ R_E$. The sensitivity to boundary conditions is clearly evident. The asymmetry in both sets of figures is due to the induced electric field resulting from time variation of the magnetosheric current systems. This is particularly noticeable in Figure 3 where without the current system the contours would be circular.

In Figure 3 the change in potential, ϕ, between the ionosphere (where it is assumed to be zero) and the equatorial plane is illustrated. It is evident from this figure that if the ionospheric electric field (and ϕ) is variable, the total electric field in the magnetoshere will also vary substantially with time. <u>Thus, through this mechanism, the ionosphere may exert some influence on the dynamics of magnetospheric fields and low energy plasma</u>. Also, Figure 2 indicates the change in ϕ between the ionosphere and the equatorial plane when the ionospheric value of ϕ is taken to be zero. <u>Thus, magnetic field lines are not equipotentials even when only the "small" induced fields like the wobbling dipole are present</u>.

The various components of the <u>total</u> electric field which would be "felt" by a charged particle rotating with the earth in geosynchronous orbit are illustrated in Figures 4 and 5. The uniformally magnetized, rotating sphere boundary condition was used for these calculations. Figure 4 displays the field as a function of longitude; Figure 5, the electric field as a function of time. Contours of constant $|\overline{E}_{total}|$ in the equatorial plane are shown in Figures 6

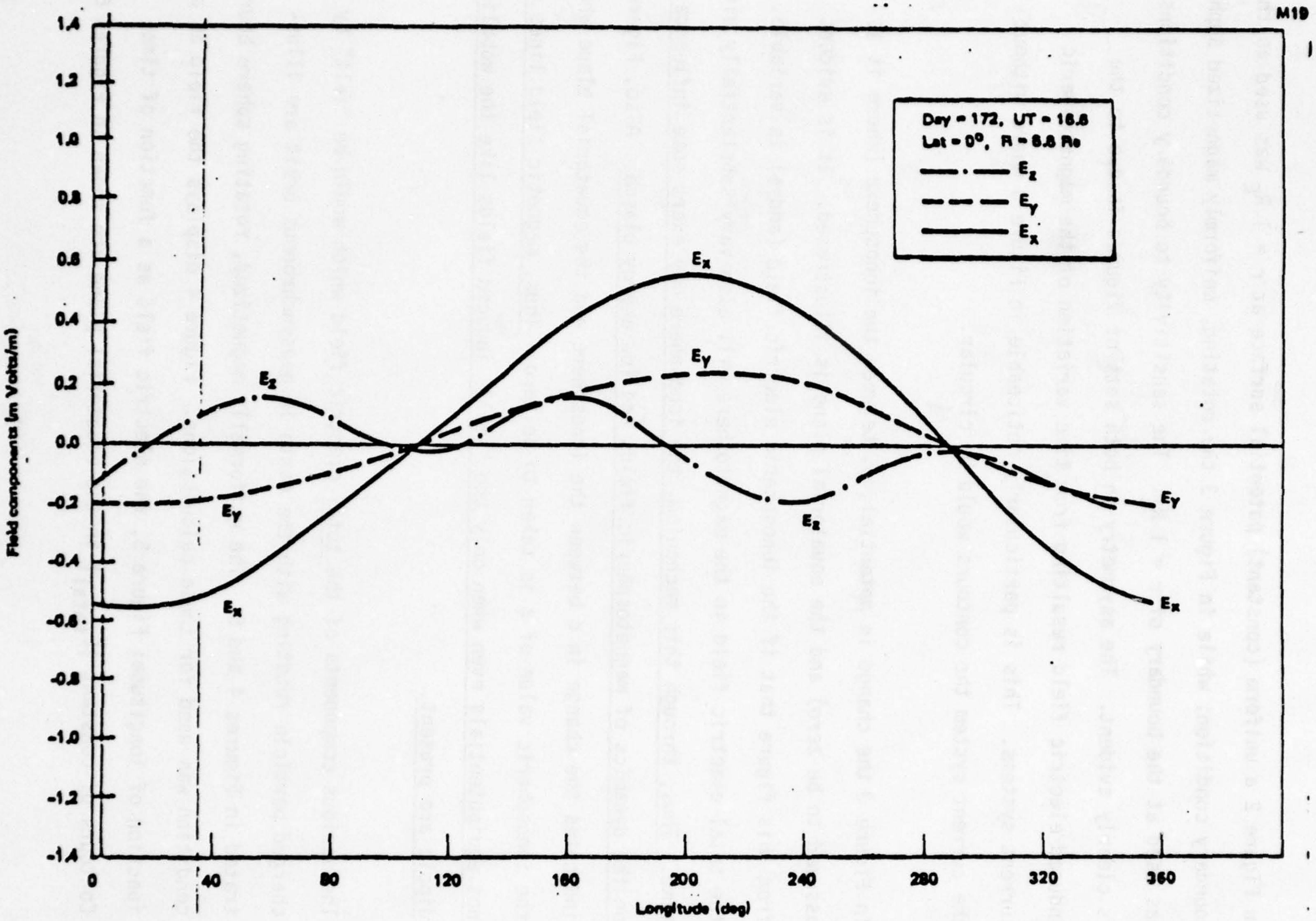


Figure 4. Total Electric Field Experienced by a Charged Particle Rotating with the Earth at Synchronous Orbit

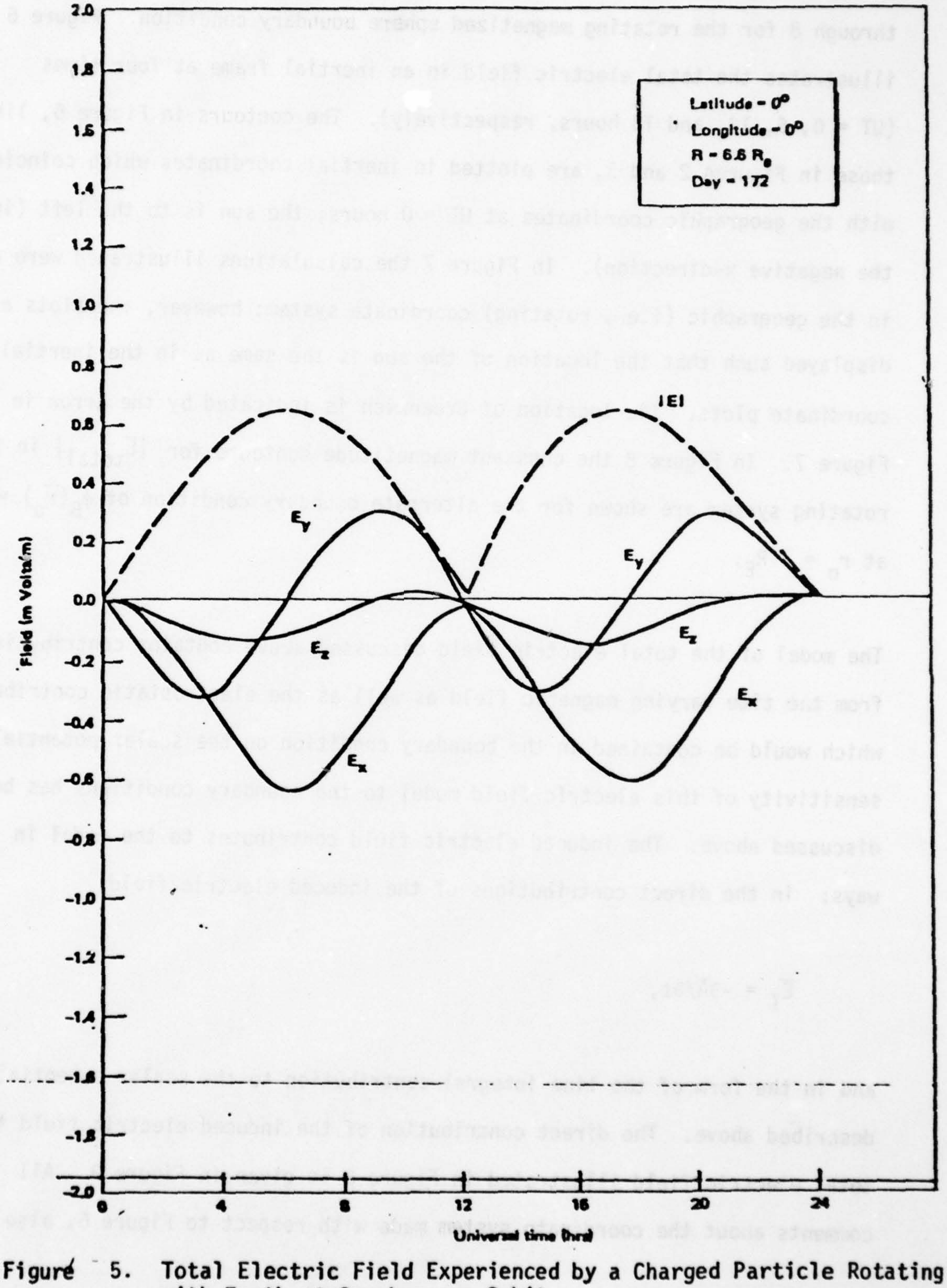


Figure 5. Total Electric Field Experienced by a Charged Particle Rotating with Earth at Synchronous Orbit

through 8 for the rotating magnetized sphere boundary condition. Figure 6 illustrates the total electric field in an inertial frame at four times (UT = 0, 6, 12, and 18 hours, respectively). The contours in Figure 6, like those in Figures 2 and 3, are plotted in inertial coordinates which coincide with the geographic coordinates at UT = 0 hours; the sun is to the left (in the negative x-direction). In Figure 7 the calculations illustrated were done in the geographic (i.e., rotating) coordinate system; however, the plots are displayed such that the location of the sun is the same as in the inertial coordinate plots. The location of Greenwich is indicated by the arrow in Figure 7. In Figure 8 the constant magnetitude contours for $|\overline{E}_{total}|$ in the rotating system are shown for the alternate boundary condition of $\phi_B(\overline{r}_o) = 0$ at $r_o = 1\ R_E$.

The model of the total electric field discussed above contains contributions from the time varying magnetic field as well as the electrostatic contributions which would be contained in the boundary condition on the scalar potential. The sensitivity of this electric field model to the boundary conditions has been discussed above. The induced electric field contributes to the model in two ways: in the direct contributions of the induced electric field,

$$\overline{E}_I = -\partial\overline{A}/\partial t,$$

and in the form of the line integral contribution to the scalar potential described above. The direct contribution of the induced electric field to the total electric field illustrated in Figure 6 is given in Figure 9. All comments about the coordinate system made with respect to Figure 6, also

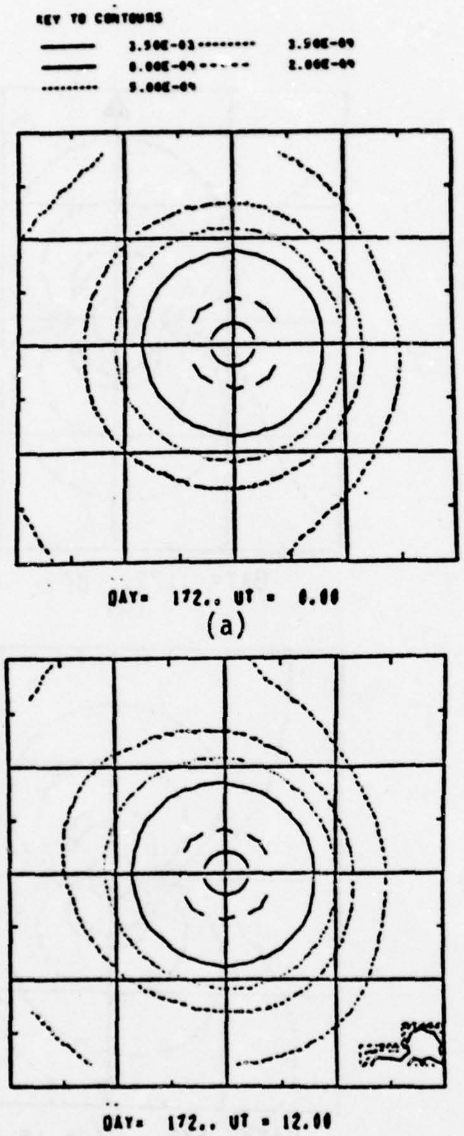


Figure 6. $|E_{total}|$ in the inertial frame, rotating magnetized sphere boundary condition. (Units are volts/meter, the scale is 5 R_E per major division.)

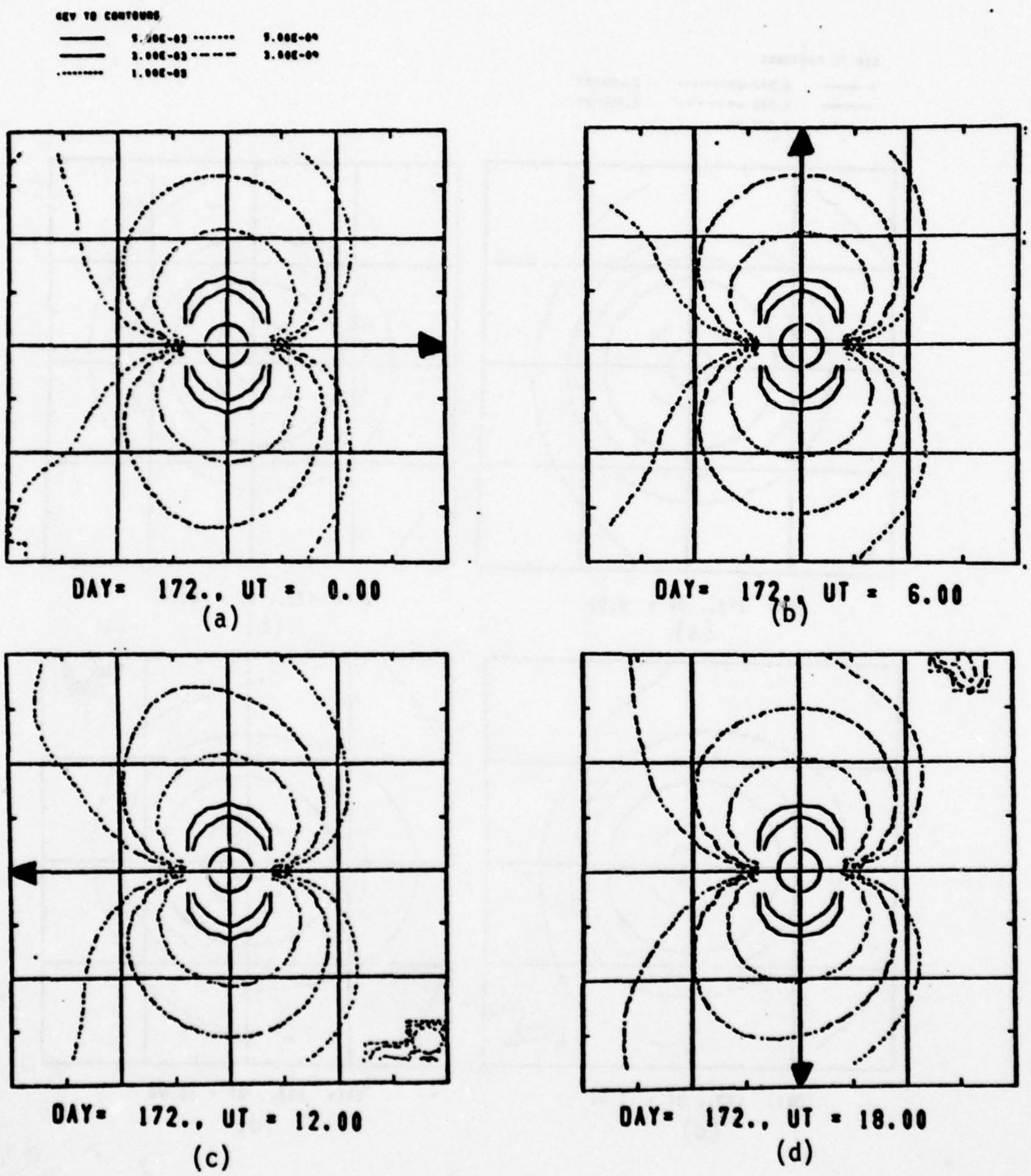


Figure 7. $|E_{total}|$ in a rotating frame for a rotating uniformally magnetized sphere boundary condition. (Units are volts/meter and the major divisions are 5 R_E.)

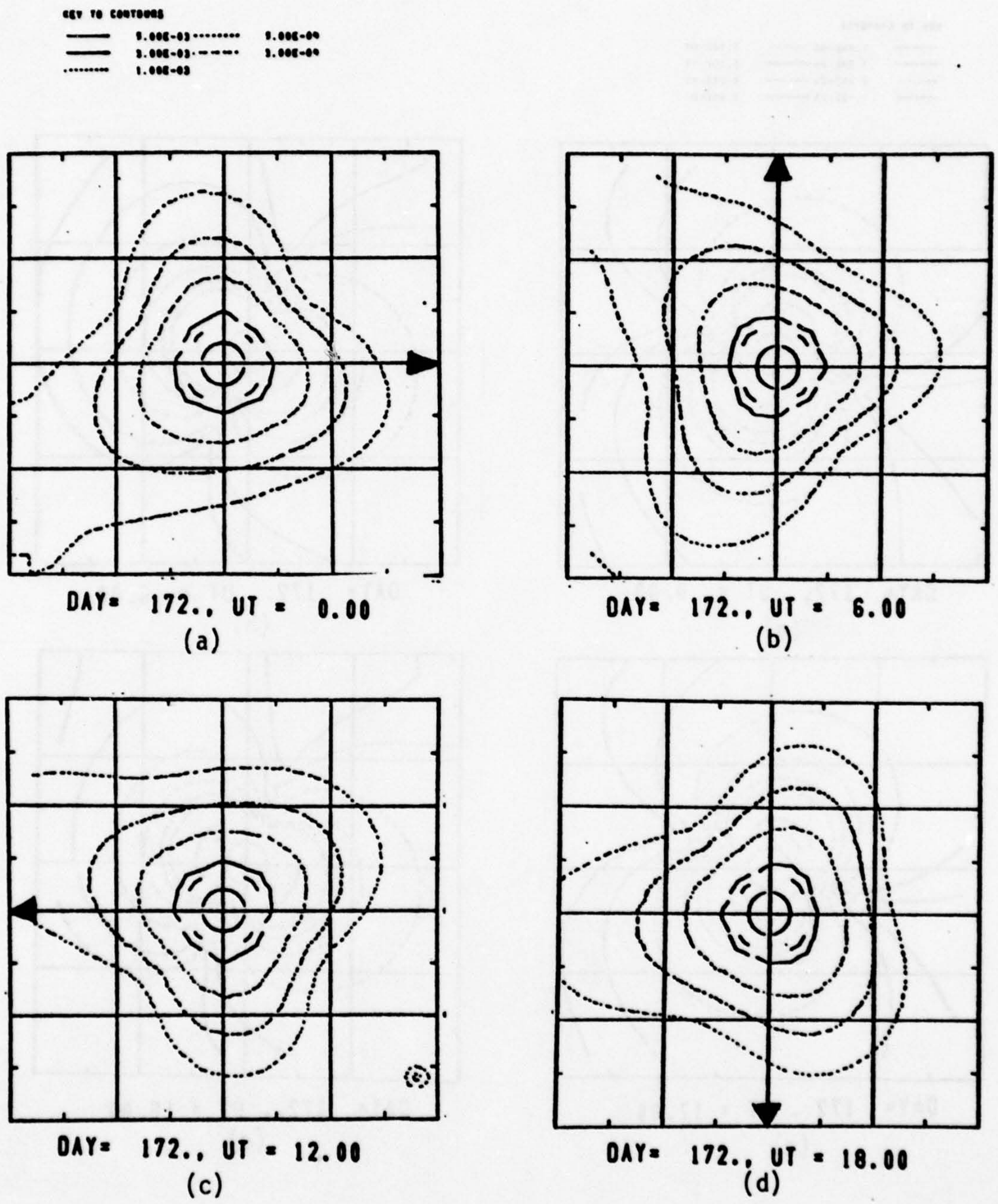


Figure 8. $|E_{total}|$ in a rotating frame for a uniform boundary condition. (Units are volts/meter and the major divisions are 5 R_E.)

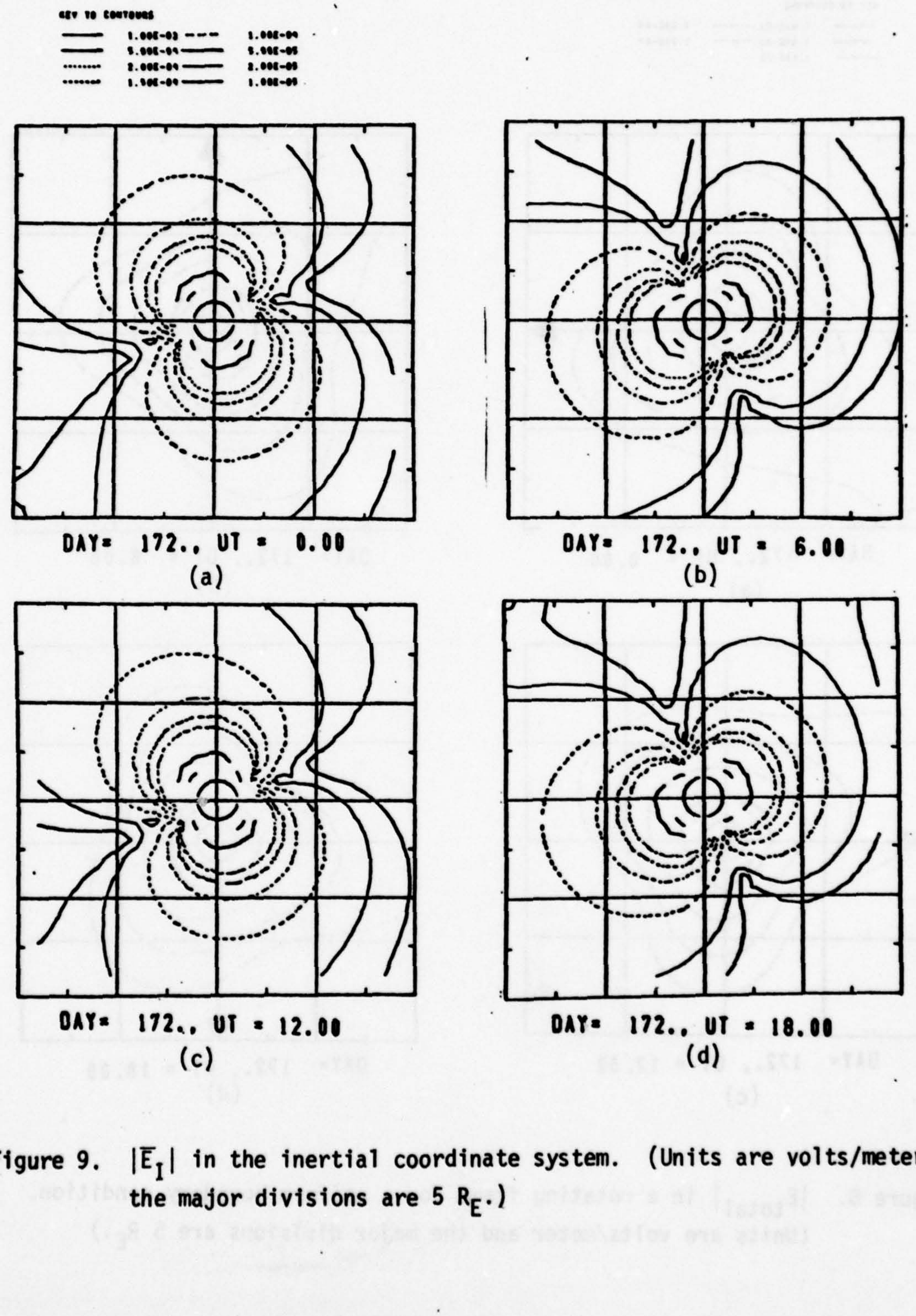


Figure 9. $|\bar{E}_I|$ in the inertial coordinate system. (Units are volts/meter, the major divisions are 5 R_E.)

apply to Figure 9. The range of values for $|\overline{E}_I|$ illustrated are 1.0 to 0.01 mV/m, although only portions of the 1 mV/m contour can be seen in the figure (the 1.0 mV/m contour is nearest the 1 R_E, earth outline). In regions where the induced field is a maximum, its direct contribution is seen to be about 14 percent of the total field. Since the induced electric field is presented in inertial coordinates, the dipole contributions are seen to dominate for small values of R. Distortions from the dipole pattern are due to the diurnal variations in the external current system (although these are also due to the motion of the dipole); these are seen to become significant at $R > 4\ R_E$ in regions of maximum $|\overline{E}_I|$, but are important over most values, R, in the region of minimum $|\overline{E}_I|$ (i.e., in the cusps).

3.3 CHARGED PARTICLE ACCELERATION

To indicate how fields such as those illustrated in Figures 4 and 5 would contribute to charged particle acceleration, the time rate of change of a proton's kinetic energy was calculated using the formula from Northrop (1963)

$$\frac{d\langle w\rangle}{dt} = q\ \overline{E}\cdot\overline{V} + M\ \frac{\partial B}{\partial t} \quad ,$$

where $\langle w\rangle$ is the kinetic energy averaged over a gyration period, M is the magnetic moment of the proton, q its charge, and $\overline{V}$ the velocity of its guiding center. The results of this calculation are shown in Figures 10 and 11. Figure 9 shows the energy gained by the guiding center of motion as a function of the guiding center kinetic energy, and similarly, Figure 10 shows the energy change in the motion <u>about</u> the guiding center.

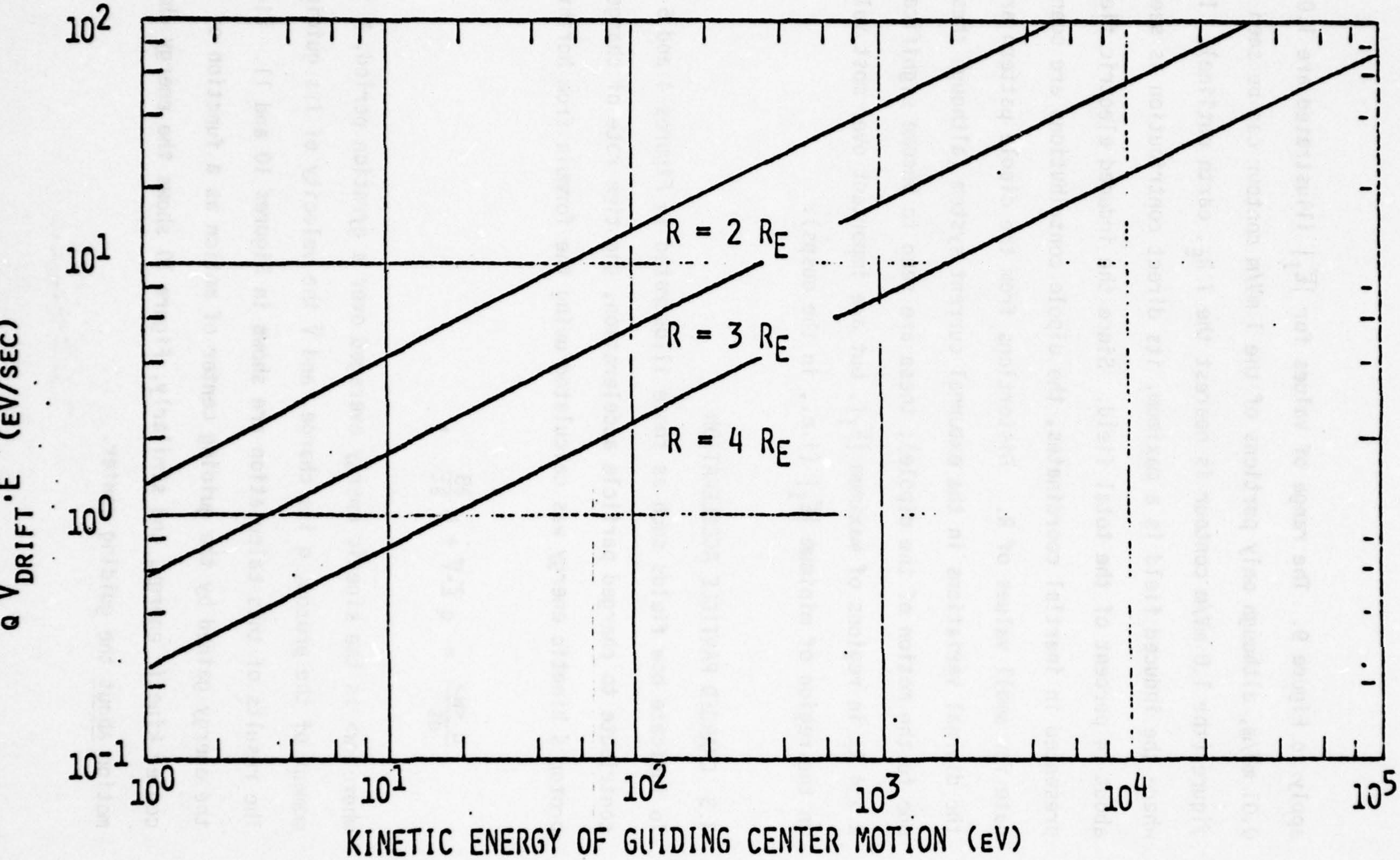


Figure 10. Time Rate of Change of Proton Kinetic Energy, Guiding Center Motion Contribution (UT = 16.6 Hrs., Day = 172, Equatorial Plane, Longitude = 0°.

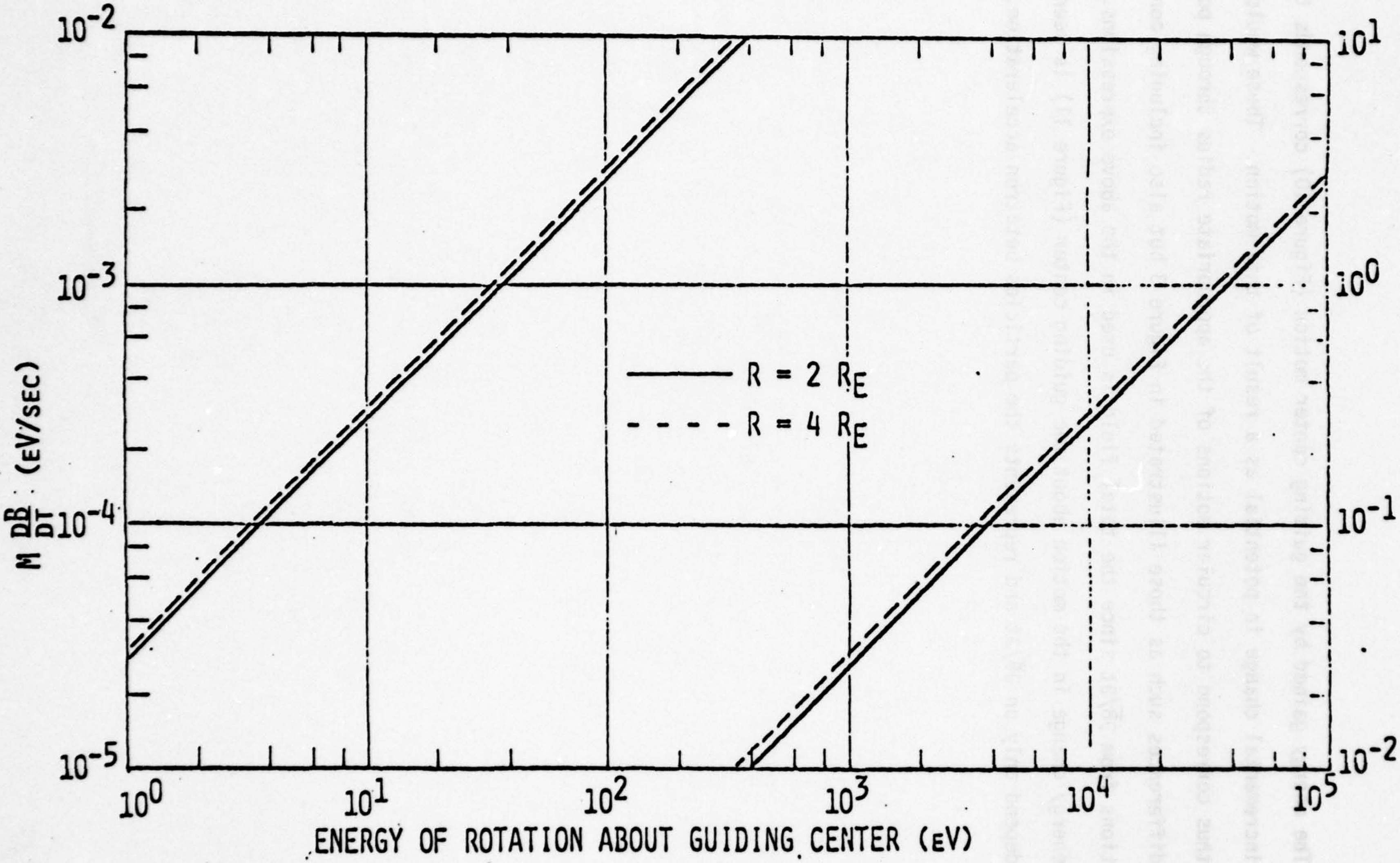


Figure 11.. Time Rate of Change of Proton Kinetic Energy Due to Rotation About the Guiding Center (UT = 16.6, Day = 172, Equatorial Plane, Longitude = 0°.

The energy gained by the guiding center motion (Figure 10) corresponds to the incremental change in potential as a result of this motion. These would thus correspond to circular motions of the appropriate radius through potential differences such as those illustrated in Figure 3 but also including contributions from $\partial\overline{B}/\partial t$ since the total field is used in the above expressions. The energy change in the motion about the guiding center (Figure 11) is seen to depend only on $\partial\overline{B}/\partial t$ and represents the particles betatron acceleration.

Section 4
SUMMARY AND CONCLUSIONS

It is an observed fact that many regions of the magnetosphere contain charged particles with energies of several tens of kilovolts. It is also known that the source of these particles is either the earth's upper atmosphere or the solar wind, both of which have particle enegies of one kilovolt (the solar wind), or only a few electron volts (the high latitude upper atmosphere). Thus, it is necessary to explain the energization of charged particles occurring locally, that is, within the magnetosphere. Only electric and time varying magnetic fields can produce this energization. It is therefore important to understand and quantitatively describe such fields in the earth's magnetosphere. Of the various sources of electric fields, the time varying magnetic field has been documented most thoroughly. In our work on magnetic field modeling we have routinely computed the vector magnetic potential simultaneously with the calculation of magnetic induction. A knowledge of the time history of the magnetic potential readily yields the associated induced electric field. Thus, the induced electric field from any time varying magnetic field can be determined if the magnetic field and its associated magnetic vector potential are known quantitatively. In the present work we have dealt primarily with the electric field induced by the daily wobble of the earth's dipole field and the associated response of the magnetospheric current systems. Such a field is important because it is always present. The results of this work suggest quantitatively the presence of an electric field that should exhibit some dependence on both local and universal time (because the form of the field is dependent on the location of

the dipole axis and magnetospheric magnetic field topology). It was found quantitatively that the raw induced electric field is on the order of a small fraction of a millivolt/meter throughout most of the magnetosphere. This suggests that the induced electric field associated with the daily wobble of the earth's magnetic field is large enough to be important to the dynamics of charged particles in the magnetosphere.

What is actually needed for the study of particle field interactions is the total electric field produced as the magnetospheric plasma responds to this induced electric field. As might be expected, the response of the plasma to this induced electric field, or any other induced electric field, depends on both the intensity of the magnetic field and various plasma parameters. Our analyses suggest that it may be necessary to treat the response differently in different regions of the magnetosphere. Generally, however, it may be argued that the plasma responds to the induced electric field by tending to short out the field in the direction of the magnetic field. This is because of the rather high conductivity that the charged particles have along magnetic field lines. The cancelling out of the induced electric field along the direction of the magnetic field is actually carried out by the setting up and maintenance of a very weak charge separation system. This electric field can be described as a divergence of a scalar potential. The problem of determining the total electric field then becomes one of finding the appropriate value of this scalar potential. This necessitates the determination of the boundary condition on the scalar potential. The study of the appropriate boundary conditions on the scalar potential is probably worthy of its own separate effort. <u>Our work on this subject to date suggests two important findings; first it is not</u>

appropriate to treat magnetic field lines as electrical equipotentials, thus the mapping of the ionospheric electric field to magnetospheric regions must involve a more detailed process. The change in potential along a magnetic field line in the presence of the electric field induced by the wobbling dipole is as large as 13 kilovolts, between the earth's surface and geosynchronous altitudes (along a field line). The other finding is based on the dependence of the electric field on the value of the scalar potential in the ionosphere. This suggests strongly that as the ionospheric electric field (and its associated scalar potential) vary appreciably as it is "driven" by these changes in the upper atmosphere and ionosphere. This suggests that some magnetospheric dynamic processes may be the result of changes occurring in the ionosphere.

A detailed examination of the total electric field produced by the wobbling dipole suggests that its impact on particle energization and acceleration may be an important feature in the vicinity of geosynchronous orbit during quiet magnetic conditions. It might be expected that this field, since it rotates somewhat in phase with the dipole, would have the largest impact on particles moving around the earth with the same angular velocity. It turns out that one kilovolt protons (the solar wind - the raw material for the magnetospheric plasma is comprised of approximately one kilovolt protons) take about one day to move around the earth at geosynchronous altitude. Thus, the induced electric field may be an important mechanism for energizing some of the low energy particles as they enter from outside of geosynchronous orbit and move radially inward through the outer Van Allen region.

The actual energization process has not yet been studied. However, it is known that this particular electric field is not conservative and can permanently energize charged particles. Also, it would be expected because of phase differences between the electric field and the charged particle motions at different locations in geosynchronous longitude, that only a fraction of the particles would actually be energized and others may lose energy. However, it is well known that even in the outer Van Allen zone the energized particles comprised only a small fraction of the total plasma density. The effect of this mechanism on charged particles then might be summarized by saying, "You can accelerate some of the particles some of the time - and that may be enough."

It is believed that this study constitutes the first quantitative examination of an electric field in the magentosphere where its source is understood quantitatively. The particular mechanism examined, that produced by the daily wobble of the dipole magnetic field and the response of the magnetosphere to it, produces an induced electric field that is present all the time, and large enought to be of interest to the study of charged particle dynamics. It is hoped that as new observational electric field data become available, that some effort be made to search for local time effects in the data. We would hope, in the near future, to do two things: first, to study the boundary conditions on the scalar potential for the electric field in more detail as it is determined by ionospheric electric fields; and, secondly, to quantitatively describe an electric field induced by time varying magnetic fields associated with the magnetospheric substorm and magnetic storm processes. Finally, the response of particles and plasma to such combined electric and magnetic fields, which have been described quantitatively

in a time varying setting, should be studied in detail. Hopefully such self consistent calculations will reveal a more detailed understanding of several dynamic magnetospheric processes.

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Appendix A

ELECTRIC FIELD CALCULATIONS

A computer program was developed to perform the electric field calculations described in this report. The key to these calculations are two subroutines (BXYZMU and AXYZ) which contain a model (Olson-Pfitzer) of the earth's external magnetic field, $\overline{B}_E$, and the associated vector potential, $\overline{A}_E$ (respectively). (The Olson-Pfitzer model is described in Reference A1, and the model for the vector potential in Appendix B of this report.)

All electric field calculations are performed in an earth centered, inertial coordinate system. Motion of the earth around the sun is important only to the extent if affects the external magnetic field (i.e., the tilt angle dependence) otherwise it is insignificant and is neglected in these calculations. Because of this choice of coordinate system, the total magnetic induction vector, $\overline{B}_T$, and vector potential, $\overline{A}_T$, must include contributions from the earth's dipole field. Thus, to the $\overline{B}_E$ and $\overline{A}_E$, obtained from subroutine BXYZMU and AXYZ, are added

$$\overline{B}_D = \frac{3\,\hat{n}\,(\hat{n}\cdot\overline{\mu}) - \overline{\mu}}{r^3}$$

and $$\overline{A}_D = \frac{\overline{\mu} \times \overline{r}}{r^3}$$

where $\overline{\mu}$ is the dipole moment

and $\hat{n} = \frac{\overline{r}}{r}$.

The induced electric field $\overline{E}_I = -\partial\overline{A}/\partial t$ is obtained by two calls to AXYZ: one at time t and the other for time t+Δt. This gives $\partial\overline{A}_E/\partial t$ which must be combined with

$$\frac{\partial \overline{A}_D}{\partial t} = \frac{(\overline{\omega} \times \overline{\mu}) \times \overline{r}}{r^3}$$

where ω is the dipoles angular velocity.

The total electric field which is generated by the temporal variations of the magnetic field is expressed by

$$\overline{E}_T = -\nabla\phi \quad \frac{\partial \overline{A}_T}{\partial t} \quad .$$

The scalar potential, , is obtained from the integral

$$\phi(r) = \phi_B - \int_{FL} \frac{\partial \overline{A}}{\partial t} \cdot \hat{e}_1 \, ds,$$

as described in this report. This integral is evaluated numerically (using the S. Gill technique [Gill, 1951]). To obtain the gradient of ϕ, $\phi(r)$ must be evaluated at four points: these are selected to be $\overline{r}$, $\overline{r} + \hat{e}_1 ds$, $\overline{r} + \delta\hat{e}_2$, $\overline{r} + \delta\hat{e}_3$, where ds is the integration step, and δ an appropriate incremental step size. The unit vector $\hat{e}_1$ is tangent to $\overline{B}_T$ at point $\overline{r}$, $\hat{e}_2$ is a unit vector parallel to the principal normal (obtained from $d\hat{e}_1/ds$), and $\hat{e}_3 = \hat{e}_1 \times \hat{e}_2$. Thus,

$$\nabla\phi \simeq \frac{\phi(\overline{r}+ds\ \hat{e}_1) - \phi(\overline{r})}{ds}\ \hat{e}_1 + \frac{\phi(\overline{r}+\delta\hat{e}_2) - \phi(\overline{r})}{\delta}\ \hat{e}_2$$

$$+ \frac{\phi(\overline{r}+\delta e_3) - \phi(\overline{r})}{\delta}\ \hat{e}_3$$

The direction of integration (i.e., of $\hat{e}_1$) is chosen so that the integral terminates at $r_o = 1\ R_E$, where a boundary value, $\phi_B = \phi(\overline{r}_o)$, is evaluated. Two options are currently available in this program

$$\phi(\overline{r}_o) = 0,$$

a uniform boundary condition, and

$$(\overline{r}_o) = \overline{A}_D(\overline{r}_o)$$

$$= \frac{(\overline{\omega}x\overline{r}_o)\quad(\overline{\mu}x\overline{r}_o)}{r_o^3}\ ,$$

the uniformally magnetized rotating sphere boundary condition.

The above procedure yields the total electric field,

$$\overline{E}_T = -\nabla\phi - \frac{\partial\overline{A}_T}{\partial t}\ ,$$

in the inertial coordinate system. A particle rotating with the earth will experience a net electromagnetic force $\overline{F} = q(\overline{E}_T + \overline{v} \times \overline{B}_T)$, where $\overline{v} = \overline{\omega}x\overline{r}$, and

q is the charge on the particle. This force transforms as a simple vector from the inertial coordinate system to a coordinate system rotating with the earth. Thus, the electric field observed by this rotating particle is given by

$$\bar{E}_{T,rot} = \frac{\bar{F}_{rot}}{q} ,$$

where both vector quantities are expressed in the rotating coordinate system.

The two magnetic field/vector potential routines, BSYZMU and AXYZ, are formulated in yet another coordinate system: The solar-magnetic coordinate system in which the Z-axis is antiparallel to $\bar{\mu}$, the earth's dipole moment, and the earth-sun direction is in the XZ-plane. This coordinate system was selected for these models (i.e., $\bar{A}_E$ and $\bar{B}_E$) because it simplifies their formulation, but it does create the necessity of numerous coordinate transformations during the calculations. For convenience, a special subroutine (COORD) was written to perfrom these transformations. Another subroutine, ANGLE, calculates the position of the sun in the celestrial sphere and determines the tilt angle (μ, used in AXYZ and BXYZMU) as well as the rotation sines and cosines for transforming from geomagnetic to solar magnetic coordinates.

Appendix B
VECTOR POTENTIAL SUBROUTINE AXYZ

A listing of the subroutine to calculate the vector potential is included in this appendix. This subroutine is not in a final ANSI standard form. It is the initial research code generated by the least squares fitting routine. Thus, the FORTRAN is not necessarily standard, efficient or easily transferable to to other machines. The code was written to run on a CDC cyber 74 using the FTN compiler.

The routine has proven very useful in calculating the induction electric field. Since the least squares routine generates the FORTRAN directly, various fitting schemes were easily evaluated.

The routine returns the vector potential in units of gauss - R_E for a given location. The coordinate system used is the solar magnetic. The calling sequence for the routine is:

Input

XX A three dimensional array of the position where the field is to be calculated.

- XX(1) is in the solar direction perpendicular to X(3).
- XX(3) is parallel to the north pointing dipole axis.
- XX(2) make a right handed coordinate system.

TILT The tilt angle of the dipole axis is to the sun earth line.

Output

AT A three dimensional output array giving the vector components of the vector potential in units of gauss - R_E

AMAG The magnitude of the vector potential.

A separate entry point to the routine, DAXYZ, calculates $\partial\overline{A}/\partial\mu$, the derivative of the vector potential with respect to the tilt angle (caution - entry points are highly machine dependent, this feature for example would not work on an IBM machine). Note that all coefficients in the listing were generated by machine so that no keypunch errors are present.

```
      SUBROUTINE AXYZ(XX,TILT,AT,AMAG)
C     THIS SUBROUTINE CALCULATES THE INDUCED ELECTRIC FIELD FROM THE
C     CURRENTS FLOWING ON THE BOUNDARY THE RING AND THE TAIL
C        XX    IS THE INPUT VARIABLE GIVING THE POINT AT WHICH THE FIELD
C              IS TO BE CALCULATED IN EARTH RADII IN SOLAR MAGNETIC COORDS
C              X(1) IS TOWARD THE SUN
C              X(3) IS PARALLEL TO THE DIPOLE AXIS
C              X(2) FORMS A RIGHT HANDED COORD. SYSTEM
C        TILT IS THE TILT ANGLE IN DEGREES
C        AT    IS A 3 DIMENSION OUTPUT VECTOR GIVING THE INDUCED FIELD
C              IN UNITS OF GAUSS-RE
C        AMAG IS THE MAGNITUDE OF THE INDUCED FIELD
C
C        DAXYZ IS AN ADDITIONAL ENTRY POINT TO THE ROUTINE TO CALCULATE
C               THE DERIVATIVE OF THE INDUCED FIELD WITH RESPECT TO THE
C              TILT ANGLE
      DIMENSION D(100),E(100),F(100),DM(200),EM(200),FM(200),DRT(200),
     *ERT(200),FRT(200),ITD(100),ITE(100),ITF(100),TT(4),AA(3)
     *,AT(3),X(10),Y(10),Z(10),XX(3)
      DATA ITD /1,2,1,2,1,1,2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1,1,2,1,2,1,1,
     *2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1/
      DATA ITE /1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,
     *1,1,1,2,1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,1,
     *1,1,2/
```

```
      SUBROUTINE AXYZ(XX,TILT,AT,AMAG)
C   THIS SUBROUTINE CALCULATES THE INDUCED ELECTRIC FIELD FROM THE
C   CURRENTS FLOWING ON THE BOUNDARY THE RING AND THE TAIL
C      XX    IS THE INPUT VARIABLE GIVING THE POINT AT WHICH THE FIELD
C            IS TO BE CALCULATED IN EARTH RADII IN SOLAR MAGNETIC COORDS
C            X(1) IS TOWARD THE SUN
C            X(3) IS PARALLEL TO THE DIPOLE AXIS
C            X(2) FORMS A RIGHT HANDED COORD. SYSTEM
C      TILT IS THE TILT ANGLE IN DEGREES
C      AT    IS A 3 DIMENSION OUTPUT VECTOR GIVING THE INDUCED FIELD
C            IN UNITS OF GAUSS-RE
C      AMAG IS THE MAGNITUDE OF THE INDUCED FIELD
C
C      DAXYZ IS AN ADDITIONAL ENTRY POINT TO THE ROUTINE TO CALCULATE
C             THE DERIVATIVE OF THE INDUCED FIELD WITH RESPECT TO THE
C             ANGLE
      DIMENSION D(100),E(100),F(100),DM(200),EM(200),FM(200),DRT(200),
     *ERT(200),FRT(200),ITD(100),ITE(100),ITF(100),TT(4),AA(3)
     *,AT(3),X(10),Y(10),Z(10),XX(3)
      DATA ITD /1,2,1,2,1,1,2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1,1,2,1,2,1,1,
     *2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1/
      DATA ITE /1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,
     *1,1,1,2,1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,1,
     *1,1,2/
```

```
      DATA ITF /2,1,2,1,2,2,1,2,2,2,1,2,1,2,2,1,1,2,1,2,2,2,2,1,2,1,2,2,
     *1,2,2,2,1,2,1,2,2,1,1,2,1,2,2,2/
      DATA (DM(I),I=1,88)/
     *-.729348268E+00,-.126711994E-03,-.250256245E-02,-.929734000E-06,
     *-.651629108E-01,-.166931408E-04,-.429000833E-03,-.274123969E-07,
     *-.313618130E-02,-.569972419E-05, .101581273E-02, .210875857E-05,
     * .100224170E-04, .447416904E-08, .157165005E-04, .371435608E-07,
     *-.302681932E-03, .526556403E-06, .139932388E-03, .163943276E-06,
     * .236910507E-05, .404562068E-09,-.174908288E-02, .111043613E-05,
     * .395551400E-05, .196463764E-07,-.172010105E-04, .104333857E-06,
     * .328361431E-05,-.381150320E-08, .188742505E-05, .796065264E-08,
     * .344430885E-06, .133982119E-08, .245745657E-03, .220385117E-06,
     * .245904885E-05, .986230049E-09, .183409240E-04, .265129601E-07,
     *-.212837026E-05,-.872199184E-08, .172169713E-04, .294076230E-08,
     *44*0./
      DATA (EM(I),I=1,128)/
     * .539534465E+01,-.917246729E-03,-.202212227E-02,-.129650634E-05,
     * .848876852E+00, .710329502E-04, .305008723E-02, .120551460E-05,
     *-.598376603E-01,-.165177282E-06,-.334958332E-02, .472213553E-05,
     * .422206531E-04, .173170587E-07, .160966776E-03, .208088996E-08,
     * .201674318E-02, .869926096E-05,-.170424338E-02,-.280163907E-05,
     *-.741748071E-05,-.919757909E-08,-.426006943E-05,-.533888339E-07,
     * .459399443E-01, .163072391E-04, .445957685E-03, .511926130E-08,
     * .562030059E-03,-.263083865E-06,-.400241568E-04,-.228038044E-06,
```

```
     *-.265641610E-05,-.173364107E-09,-.111586849E-03, .487350690E-07,
     * .558513796E-06,-.355505259E-09,-.173441325E-04,-.139197938E-08,
     * .149384635E-05,-.184142610E-08,-.565550702E-03,-.127131636E-05,
     *-.377826457E-05,-.241832155E-07, .395029200E-04,-.150796074E-06,
     * .141863427E-04, .157547770E-07,-.894741777E-05, .441400372E-07,
     * .869472657E-05,-.170170836E-07,-.123151650E-06,-.675272345E-10,
     *-.549566250E-04,-.134417266E-07, .538339684E-05, .119398067E-07,
     *-.105866429E-03,-.224420771E-06,-.298911521E-05,-.432303236E-09,
     *64*0./
      DATA (FM(I),I=1,88)/
     * .452063859E-02, .309321235E-06, .658118267E-01, .922543691E-05,
     * .221930272E-03,-.269376799E-06, .434366116E-02,-.169477296E-05,
     * .264268049E-03,-.496539062E-07,-.383052388E-04,-.818745994E-08,
     *-.139054930E-03,-.902753036E-07,-.122044254E-05, .305995114E-09,
     * .299405601E-05,-.823980252E-08,-.137081926E-06, .686745107E-09,
     *-.449407954E-05, .516272489E-08,-.108807156E-03,-.225109712E-07,
     *-.387313966E-03,-.181461377E-06,-.257814468E-05, .362172735E-09,
     * .107887529E-05,-.817801019E-10, .327270756E-04,-.213768490E-06,
     * .182395159E-04,-.467691071E-07,-.425729821E-05, .457564728E-08,
     *-.507417182E-04, .234472682E-07,-.828838728E-06,-.222432842E-09,
     * .118487595E-06, .259558968E-10, .445023379E-06, .345792292E-09,
     *44*0./
      DATA (DRT(I),I=1,76)/
     * .172710258E+02, .901487514E-04,-.260481734E-02,-.295958214E-06,
```

```
*-.145645931E+00, .254212737E-04, .383375497E-03, .293137137E-07,
*-.164998210E+00, .109790856E-05,-.114098254E+00,-.145363986E-05,
* .421889326E-05, .163566423E-09, .690795134E-03, .108377673E-07,
* .849415447E-05,-.882774765E-07,-.666521735E-03,-.123262817E-06,
*-.107038820E-06, .183526004E-09,-.665902459E-01,-.614480180E-05,
*-.620774007E-05, .677287959E-08, .271871068E-03, .132535235E-07,
* .371257126E-03, .103666350E-07,-.301493635E-05,-.314695777E-08,
*-.499745073E-06,-.271292714E-09,-.688605770E-04, .959489100E-07,
*-.165221045E-05,-.107388026E-08, .515588023E-03, .364233560E-08,
* .243895926E-03, .422718108E-08,-.488999251E-04, .300077464E-07,
*-.864493180E+01,-.137564157E-03, .885969499E-03, .111255015E-06,
* .951227893E-01,-.530732966E-05, .611010433E-03, .151901631E-06,
* .150255927E+01, .217273338E-04, .152177226E+01,-.662891287E-05,
*-.107720777E-03,-.342878767E-08,-.524925990E-01,-.132600472E-05,
* .395262265E-01,-.286208418E-05,-.589201109E-01, .106880661E-05,
*-.861750112E-04,-.270153135E-07, .175316143E+01,-.118241166E-04,
*-.190901686E-03,-.723388430E-07,-.612874648E-01,-.688835429E-06,
*-.220901978E-01, .937944858E-06, .380989611E-04, .134575158E-08/
 DATA (DRT(I),I=77,88)/
*-.417001831E-04,-.151863693E-08,-.664028370E-01, .113662696E-05,
* .616193982E-05,-.666390209E-08,-.530383681E-01,-.975302396E-06,
*-.748297216E-02,-.219752436E-06,-.239583014E-01, .161912657E-07/
 DATA (ERT(I),I=1,76)/
* .223574068E+02, .237092468E-02,-.259265244E-01,-.543297508E-05,
```

```
*-.203285835E+02, .372970371E-03, .861737775E-02, .731381800E-06,
*-.192862450E+00,-.987028865E-05, .259278543E+00,-.283607527E-04,
* .369239478E-05,-.374637485E-07,-.295726213E-02, .181854045E-06,
* .221685668E+00,-.166140170E-04, .583366561E-01,-.627761240E-05,
*-.180559377E-04, .555974814E-08,-.179696137E-03, .520626763E-07,
* .895745381E+00,-.674934702E-04,-.986488524E-03,-.275597128E-06,
*-.418416981E-02, .129011212E-05,-.191967792E-02, .334511131E-06,
* .242243391E-05, .970809045E-09, .272726317E-03, .201624589E-07,
* .370105623E-06, .436422359E-09,-.367325152E-04, .125324253E-07,
* .192694269E-06, .937252238E-09, .629491827E-01, .161911909E-05,
* .434586625E-04,-.565854074E-08,-.844541996E-03, .297073369E-07,
*-.183302340E-03, .393191421E-07,-.102564188E-02, .566193608E-07,
*-.976549253E-03, .809544992E-07, .171195945E-07, .301303214E-09,
*-.584576073E-03, .349309619E-07,-.162984387E-04, .287451325E-07,
*-.323777003E-02,-.166478703E-06, .109434413E-05, .304030761E-08,
* .363156637E+02,-.365860876E-02,-.178559220E-01, .268064055E-04,
* .341857131E+01,-.202749458E-03,-.665289424E-02,-.479999608E-05,
* .419388255E+00,-.144550505E-03,-.802490713E+00, .120394801E-03/
 DATA (ERT(I),I=77,128)/
* .176629350E-02,-.362697748E-05, .131833943E+00,-.109745454E-04,
*-.513473466E+00, .951314318E-05,-.164399232E+01,-.345042218E-04,
* .555130138E-03, .417071452E-06, .156573384E-01,-.664348824E-06,
*-.128316301E+01, .943190943E-04, .315836305E-02,-.296679057E-05,
* .119578246E+00,-.205993234E-05, .229594643E+00,-.294438385E-04,
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*-.169840264E-03, .166610251E-06, .725218345E-04,-.110254181E-05,
*-.118523999E-03, .105078925E-06, .269856625E-03, .219626732E-06,
* .115948360E-04, .282992628E-07,-.128509474E+01,-.111032723E-04,
* .650072208E-03, .638410474E-06,-.461053108E-02,-.426555486E-05,
* .947043586E-02, .231426410E-06, .352982119E-01,-.101259718E-05,
* .919490472E-01,-.132306508E-04,-.134746021E-05, .628641224E-07,
* .169132530E-01,-.323945908E-06, .201617420E-01, .120818427E-05,
* .146883633E+00,-.142977156E-04,-.204670532E-03, .812572280E-07/
 DATA (FRT(I),I=1,76)/
* .146743766E-01, .690303858E-05, .372156538E+00,-.264559434E-04,
*-.294057241E-02,-.102776737E-05,-.327854379E-01, .841716820E-05,
*-.299658270E-03,-.788783701E-07,-.104994423E-03,-.547459860E-07,
* .399988625E-03, .686099592E-07, .602133083E-06, .235828862E-09,
* .253961040E-04,-.109861910E-08, .102364669E-04, .490368829E-08,
*-.200502940E-03,-.173592698E-07, .376290963E-03, .773096003E-07,
*-.111120579E-02,-.466111470E-06, .289692070E-06, .132191930E-08,
*-.117177175E-05, .384449408E-11,-.156488755E-03, .113649545E-06,
*-.405757149E-04,-.157793338E-07,-.161509888E-04, .632240329E-08,
* .422877710E-03,-.497223441E-07, .359951195E-06, .226223418E-09,
* .372886769E-06, .179780627E-09,-.140183587E-05,-.169167210E-08,
*-.771520980E-02,-.383260698E-05,-.528815760E+00,-.414401986E-05,
* .691315582E-03, .117634964E-05,-.475324359E-01, .573775127E-05,
* .385444465E-03,-.531578937E-07, .174012946E-03, .180575867E-06,
* .737612283E-01, .390951456E-06,-.257447966E-04,-.694245701E-08,
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     * .145042963E-03, .146067293E-06,-.318832202E-04, .607128391E-07,
     * .776422462E-02,-.162131526E-05, .135424063E-03,-.517778332E-06,
     * .827816211E-01,-.135826726E-05,-.166114493E-03,-.768702465E-07,
     *-.765480102E-04,-.416505966E-07,-.118520391E-03, .100688467E-05/
      DATA (FRT(I),I=77,88)/
     * .224003816E-02,-.148649869E-05,-.126513028E-03,-.202752949E-06,
     *-.516082680E-02, .132101131E-05,-.115795115E-04,-.802893957E-09,
     *-.177513253E-04,-.170179817E-07, .578165098E-05, .567716018E-07/
      DATA TILTL/99./
      TT(1)=1.
      TT(2)=TILT
      TT(3)=TILT*TILT
      TT(4)=TILT*TT(3)
      GO TO 200
      ENTRY DAXYZ
      TILTL=TILT
      TT(1)=0.
      TT(2)=1.
      TT(3)=2.*TILT
      TT(4)=3.*TILT**2
  200 CONTINUE
      DO 10 I=1,64
      II=(I-1)*2+1
      K=ITD(I)
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      D(I)=(10.*DM(II)+DRT(II))*TT(K)+(10.*DM(II+1)+DRT(II+1))*TT(K+2)
      K=ITE(I)
      E(I)=(10.*EM(II)+ERT(II))*TT(K)+(10.*EM(II+1)+ERT(II+1))*TT(K+2)
      K=ITF(I)
10    F(I)=(10.*FM(II)+FRT(II))*TT(K)+(10.*FM(II+1)+FRT(II+1))*TT(K+2)
20    CONTINUE
      XN=XX(1)
      YN=XX(2)
      ZN=XX(3)
      R2=XN**2+YN**2+ZN**2
      R=SQRT(R2)
      DO 1 I=1,7
      X(I)=XN
      Y(I)=YN
      Z(I)=ZN
      XN=XN*XX(1)
      YN=YN*XX(2)
      ZN=ZN*XX(3)
1     CONTINUE
      AA(1)=+D( 1)*Y( 1)+D( 2)*Y( 1)*Z( 1)+D( 3)*X( 1)*Y( 1)+D( 4)*X( 1)
     **Y( 1)*Z( 1)+D( 5)*Y( 1)*Z( 2)+D( 6)*Y( 3)+D( 7)*Y( 3)*Z( 1)+D( 8)
     **Y( 3)*Z( 2)+D( 9)*X( 1)*Y( 1)*Z( 2)+D(10)*X( 1)*Y( 3)+D(11)*X( 1)
     **Y( 3)*Z( 1)+D(12)*X( 2)*Y( 1)+D(13)*X( 2)*Y( 1)*Z( 1)+D(14)*X( 2)
     **Y( 1)*Z( 2)+D(15)*X( 2)*Y( 3)+D(16)*Y( 1)*Z( 3)+D(17)*X( 1)*Y( 1)
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**Z( 3)+D(18)*X( 3)*Y( 1)+D(19)*X( 3)*Y( 1)*Z( 1)+D(20)*Y( 1)*Z( 4)
*+D(21)*Y( 5)+D(22)*X( 4)*Y( 1)
 AA(1)=AA(1)                   +(0.0 +D(23)*Y( 1)+D(24)*Y( 1)*Z( 1)
*+D(25)*X( 1)*Y( 1)+D(26)*X( 1)*Y( 1)*Z( 1)+D(27)*Y( 1)*Z( 2)+D(28)
**Y( 3)+D(29)*Y( 3)*Z( 1)+D(30)*Y( 3)*Z( 2)+D(31)*X( 1)*Y( 1)*Z( 2)
*+D(32)*X( 1)*Y( 3)+D(33)*X( 1)*Y( 3)*Z( 1)+D(34)*X( 2)*Y( 1)+D(35)
**X( 2)*Y( 1)*Z( 1)+D(36)*X( 2)*Y( 1)*Z( 2)+D(37)*X( 2)*Y( 3)+D(38)
**Y( 1)*Z( 3)+D(39)*X( 1)*Y( 1)*Z( 3)+D(40)*X( 3)*Y( 1)+D(41)*X( 3)
**Y( 1)*Z( 1)+D(42)*Y( 1)*Z( 4)+D(43)*Y( 5)+D(44)*X( 4)*Y( 1))*EXP
*( -.06*R2)
 AA(2)=+E( 1)+E( 2)*Z( 1)+E( 3)*X( 1)+E( 4)*X( 1)*Z( 1)+E( 5)*Z( 2)
*+E( 6)*Y( 2)+E( 7)*Y( 2)*Z( 1)+E( 8)*Y( 2)*Z( 2)+E( 9)*X( 1)*Z( 2)
*+E(10)*X( 1)*Y( 2)+E(11)*X( 1)*Y( 2)*Z( 1)+E(12)*X( 1)*Y( 2)*Z( 2)
*+E(13)*X( 2)+E(14)*X( 2)*Z( 1)+E(15)*X( 2)*Z( 2)+E(16)*X( 2)*Y( 2)
*+E(17)*X( 2)*Y( 2)*Z( 1)+E(18)*Z( 3)+E(19)*Y( 2)*Z( 3)+E(20)*X( 1)
**Z( 3)+E(21)*X( 2)*Z( 3)+E(22)*X( 3)+E(23)*X( 3)*Z( 1)+E(24)*X( 3)
**Z( 2)+E(25)*X( 3)*Y( 2)+E(26)*Z( 4)+E(27)*Y( 4)+E(28)*Y( 4)*Z( 1)
*+E(29)*X( 1)*Z( 4)+E(30)*X( 1)*Y( 4)+E(31)*X( 4)+E(32)*X( 4)*Z( 1)
 AA(2)=AA(2)
*+(0.0 +E(33)+E(34)*Z( 1)+E(35)*X( 1)+E(36)*X( 1)*Z( 1)+E(37)*Z( 2)
*+E(38)*Y( 2)+E(39)*Y( 2)*Z( 1)+E(40)*Y( 2)*Z( 2)+E(41)*X( 1)*Z( 2)
*+E(42)*X( 1)*Y( 2)+E(43)*X( 1)*Y( 2)*Z( 1)+E(44)*X( 1)*Y( 2)*Z( 2)
*+E(45)*X( 2)+E(46)*X( 2)*Z( 1)+E(47)*X( 2)*Z( 2)+E(48)*X( 2)*Y( 2)
*+E(49)*X( 2)*Y( 2)*Z( 1)+E(50)*Z( 3)+E(51)*Y( 2)*Z( 3)+E(52)*X( 1)
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**Z( 3)+E(53)*X( 2)*Z( 3)+E(54)*X( 3)+E(55)*X( 3)*Z( 1)+E(56)*X( 3)
**Z( 2)+E(57)*X( 3)*Y( 2)+E(58)*Z( 4)+E(59)*Y( 4)+E(60)*Y( 4)*Z( 1)
*+E(61)*X( 1)*Z( 4)+E(62)*X( 1)*Y( 4)+E(63)*X( 4)+E(64)*X( 4)*Z( 1)
*)*EXP ( -.06*R2)
 AA(3)=+F( 1)*Y( 1)+F( 2)*Y( 1)*Z( 1)+F( 3)*X( 1)*Y( 1)+F( 4)*X( 1)
**Y( 1)*Z( 1)+F( 5)*Y( 1)*Z( 2)+F( 6)*Y( 3)+F( 7)*Y( 3)*Z( 1)+F( 8)
**Y( 3)*Z( 2)+F( 9)*X( 1)*Y( 1)*Z( 2)+F(10)*X( 1)*Y( 3)+F(11)*X( 1)
**Y( 3)*Z( 1)+F(12)*X( 2)*Y( 1)+F(13)*X( 2)*Y( 1)*Z( 1)+F(14)*X( 2)
**Y( 1)*Z( 2)+F(15)*X( 2)*Y( 3)+F(16)*Y( 1)*Z( 3)+F(17)*X( 1)*Y( 1)
**Z( 3)+F(18)*X( 3)*Y( 1)+F(19)*X( 3)*Y( 1)*Z( 1)+F(20)*Y( 1)*Z( 4)
*+F(21)*Y( 5)+F(22)*X( 4)*Y( 1)
 AA(3)=AA(3)                 +(0.0 +F(23)*Y( 1)+F(24)*Y( 1)*Z( 1)
*+F(25)*X( 1)*Y( 1)+F(26)*X( 1)*Y( 1)*Z( 1)+F(27)*Y( 1)*Z( 2)+F(28)
**Y( 3)+F(29)*Y( 3)*Z( 1)+F(30)*Y( 3)*Z( 2)+F(31)*X( 1)*Y( 1)*Z( 2)
*+F(32)*X( 1)*Y( 3)+F(33)*X( 1)*Y( 3)*Z( 1)+F(34)*X( 2)*Y( 1)+F(35)
**X( 2)*Y( 1)*Z( 1)+F(36)*X( 2)*Y( 1)*Z( 2)+F(37)*X( 2)*Y( 3)+F(38)
**Y( 1)*Z( 3)+F(39)*X( 1)*Y( 1)*Z( 3)+F(40)*X( 3)*Y( 1)+F(41)*X( 3)
**Y( 1)*Z( 1)+F(42)*Y( 1)*Z( 4)+F(43)*Y( 5)+F(44)*X( 4)*Y( 1))*EXP
*( -.06*R2)
 AR=-31000./(R2*R2*R)
 AT(1)=AA(1)*0.00001
 AT(2)=AA(2)*0.00001
 AT(3)=AA(3)*0.00001
 AMAG=SQRT(AT(1)**2+AT(2)**2+AT(3)**2)


  RETURN
  END
```